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Potential Driver for Accelerating Adiabatic Dynamics in Complex Wave Functions

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ABSTRACT

This research is a theoretical study that investigates the literature reviewing the adiabatic acceleration methods of quantum particle dynamics through the application of the fast forward method. This method was initially developed by Masuda and Nakamura in 2010. In this approach, the acceleration scheme is generated through the modification of the Hamiltonian by including regularization terms in the original Hamiltonian. In this study, the fast forward method is applied to a harmonic oscillator system. The fast forward method is first employed to obtain the adiabatic phase. In the subsequent step, through a review of the system's states, a driving potential is identified that ensures the harmonic oscillator system can transition from the initial state to the final state in a short period of time while preserving the system's energy. This research employs adiabatic parameters that are of a general nature, allowing for the use of freely selectable parameters.

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1. INTRODUCTION

In this era of globalization, circumstances require us to adapt to the changing times. Science and technology (IPTEK) are rapidly advancing. The development of science and technology serves as an indicator of the progress of the times. Such as in the manufacturing process of products (e.g., electronics, automotive, factories, etc.) (Del Campo& Kim, 2019; Masuda & Nakamura, 2008) in the context of manufacturing such products, thereexists a crucial need toexpedite the production process (Benggadinda & Setiawan, 2021; Setiawan, 2019).

Manipulating and optimizing the production timeof a product is a strategy that can be implemented to reduce the duration of the product design (Benggadinda & Setiawan, 2021). Currently, a method is being developed to accelerate the dynamics of quantum particles. In the process of accelerating the dynamics of quantum particles, a formula is highly needed to shorten the time when quantum particles transition without altering their characteristics.

There is research that is developing the concept of accelerating dynamics at the microscopic scale while maintaining the characteristic energy levels of the system, known as adiabatic quantum dynamics. One method used to accelerate the dynamics of quantum particles is known as the fast forward method, which involves arranging or producing events by reducing thetime scale, similar to fast-forwarding a projected film on a screen (Nakamura et al., 2017). Researchers have also developed a theory aimed at speeding up the time in the process of quantum particle dynamics, known as the theory of "shortcuts to adiabaticity." (Berry, 2009). Various studies on quantum particle dynamics have been conducted with the aim of achieving the final state in a shorter time without altering the characteristics of the quantum particles.

Previous research has developed the fast forward method in adiabatic quantum systems (Masuda & Nakamura, 2010). In the phenomena of particle interactions and Bose gases, the fast forward method is used toobtain desired characteristics in a shorter time by determining the driving potential beforehand. Additionally, the fast forward method is also applied to quantum spin systems (Setiawan et al., 2017). In the context of quantum discussions, the time-dependent Schrödinger equation with varying mass has attracted significant attention due to its diverse applications in physics research, particularly in quantum systems involving N particles (Griffiths, 2004).

In the context of quantum systems, the harmonic oscillator problem plays a crucial role as it is often adopted as an approximate model in many cases (Fitzpatrick, 2015). This research aims toobtain an additional potential in the system so that quantum particles can move in a short time while still maintaining their characteristics in an eigenstate. In this research, the fast forward method will be investigated on complex wave functions using an arbitrary adiabatic parameter R toobtain the adiabatic phase and additional potential.

2. METHOD

This research is a theoretical study involving a literature review. The literature review is a process to gain a comprehensive understanding of previous research related to a specific topic, with the aim of establishing a foundation or rationale for theongoing research or as a sourceof inspiration for future research. This research was conducted through the method of collecting literature data involving various sources, including journals and books. After determining the topic to be investigated, the formulation of the problem is established as the focus of the research, which will then be thoroughly examined toobtain theexpected results.

3. RESULTS AND DISCUSSION

The harmonic oscillator is an important component in quantum mechanics that is often used as an approximation model to solve physically complex cases. The primary function of the harmonic oscillator is to serve as an energy source used to describe particles in the quantum domain analogously. This research focuses on solving the Schrödinger equation for particles subjected to an additional potential influence in the harmonic oscillator system. The described solutions of the Schrödinger equation are in a time-independent form.

$$\varphi\ \ (x,t) = \Phi_n(x,t)\mathrm{e}^{\frac{-\mathrm{i}}{\hbar}\int_0^{\mathrm{t}}\mathrm{E_n}(\mathrm{R}(t))\,\mathrm{dt}}\mathrm{e}^{\mathrm{i}\theta(\mathrm{x,t})} \tag{1}$$

If we consider the adiabatic wave function, equation (1) can be rewritten, where R is an arbitrary parameter and θ is the adiabatic phase.

$$\varphi^{ad}\left(x,R(t)\right) = \Phi_n\left(x,R(t)\right)e^{\frac{-i}{\hbar}\int_0^t E_n(\mathrm{R}(t))\,dt}e^{i\theta(x,t)} \tag{2}$$

The Schrödinger equation for an adiabatic wave function can be written as follows:

$$i\hbar\,\frac{\partial \varphi^{ad}}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\,\varphi^{ad} + (V+v)\varphi^{ad} \tag{3}$$

With

$$\frac{\partial \varphi}{\partial t} = \frac{\partial \varphi}{\partial R}\frac{\partial R}{\partial t} \tag{4}$$

Equation (3) is first derived on the left-hand side.

$$i\hbar\frac{\partial \Psi_{(x,R(t))}}{\partial t} = i\hbar\left[\frac{\partial \Phi_{n(x,R_{(t)})}}{\partial R}\,e^{\frac{-i}{\hbar}\int_0^t E_n(\mathrm{R}(t))\,dt}e^{i\theta(x,t)}\right]\dot{R} \tag{5}$$

Then, weobtai

$$i\hbar\,\frac{\partial \phi_\mathrm{n}}{\partial \mathrm{R}}\dot{R}\,\mathrm{e}^{\frac{-\imath}{\hbar}\int_0^t \mathrm{E_n(R}(t))\,\mathrm{dt}}\mathrm{e}^{\imath\theta(x,t)} - \hbar\,\frac{\mathrm{d}\theta}{\mathrm{dt}}\phi_\mathrm{n}\,\mathrm{e}^{\frac{-i}{\hbar}\int_0^t \mathrm{E_n(R}(t))\,\mathrm{dt}}\mathrm{e}^{i\theta(x,t)} \tag{6}$$

Next, we seek the derivativeof the right-hand sideof equation (3), which is:

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2\phi_{\mathrm{n}(\mathrm{x,R(t)})}}{\partial x^2}\mathrm{e}^{\frac{-i}{\hbar}\int_0^t \mathrm{E_n(R}(t))\,\mathrm{dt}}\mathrm{e}^{i\theta(x,t)} + 2i\,\frac{\partial \phi_\mathrm{n}}{\partial x}\frac{d\theta}{dx}\mathrm{e}^{\frac{-i}{\hbar}\int_0^t \mathrm{E_n(R}(t))\,\mathrm{dt}}\mathrm{e}^{i\theta(x,t)} + i\,\phi_\mathrm{n}\frac{\partial^2\theta}{\partial x^2} - \left(\frac{d\theta}{dx}^2\right)\phi_\mathrm{n}\,\mathrm{e}^{\frac{-i}{\hbar}\int_0^t \mathrm{E_n(R}(t))\,\mathrm{dt}}\mathrm{e}^{i\theta(x,t)}\right] \tag{7}$$

By substituting equations (6) and (7) intoequation (3), weobtain:

$$i\hbar\,\frac{\partial \phi_n}{\partial R}\dot{R} - \hbar\,\frac{d\theta}{dt}\phi_n\big(x,R(t)\big) = -\frac{\hbar^2}{2m}\,2i\,\frac{\partial \phi_n}{\partial x}\frac{d\theta}{dx} + i\,\phi_n\big(x,R(t)\big)\frac{\partial^2\theta}{\partial x^2} - \left(\frac{d\theta}{dx}\right)^2\phi_n\big(x,R(t)\big) + \tilde{v}\,\phi_n\big(x,R(t)\big) \tag{8}$$

Then, equation (8) is multiplied by $\frac{i}{\hbar}\Phi_n{}^*$ resulting in:

$$-\dot{R}\,\phi_n{}^*\frac{\partial \phi_n}{\partial R} - i\frac{d\theta}{dt}|\phi_n|^2 = -\frac{\hbar}{2m}\;2\,\phi_n{}^*\frac{\partial \phi_n}{\partial x}\frac{d\theta}{dx} - \frac{\hbar}{2m}|\phi_n|^2\frac{\partial^2\theta}{\partial x^2} - \frac{i\hbar}{2m}|\phi_n|^2\left(\frac{d\theta}{dx}\right)^2 + \frac{i\hbar}{2m}|\phi_n|^2\tilde{v} \tag{9}$$

Equation (9) is separated into its real and imaginary parts, yielding:

$$\frac{2m}{\hbar}\dot{R}\ Re\left[\phi_n\ \ \frac{\partial\phi_n}{\partial R}\right] + 2\ Re\left[{\phi_n}^*\frac{\partial\phi_n}{\partial x}\right]\frac{d\theta}{dx} + \ |\phi_n|^2\frac{\partial^2\theta}{\partial x^2} = 0 \tag{10}$$

$$\frac{\hbar}{m}\ Im\ \left[{\phi_n}^*\frac{\partial\phi_n}{\partial x}\right]\frac{d\theta}{dx} + \frac{\tilde{v}}{\hbar}|\phi_n|^2 + Im\ {\phi_n}^*\frac{\partial\phi_n}{\partial R} + \ \frac{d\theta}{dt}|\phi_n|^2 + \frac{\hbar}{2m}|\phi_n|^2\left(\frac{d\theta}{dx}\right)^2 \ = 0 \tag{11}$$

Equation (10) is theequation for determining the adiabatic phase θ, and equation (11) is used toobtain the additional potential (Masuda & Nakamura, 2010). Since the wave function under consideration is a real function, theequation to be used is equation (10).

3.1. Adiabatic Phase in Complex Wave Function

The complex wave function of the harmonic oscillator in the*ground state* is expressed as: (Griffiths, 2005).

$$\Psi_{0(x,R(t))} = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} e^{-\frac{m\omega}{2\hbar}x^2} e^{i\eta} e^{i\theta(x,t)} \tag{12}$$

With

$$\eta = B\int_0^x exp\left[\frac{m\omega}{\hbar}x^2\right]\ dx \tag{13}$$

The graph of the complex wave function of the harmonic oscillator can be observed in Figure 1.

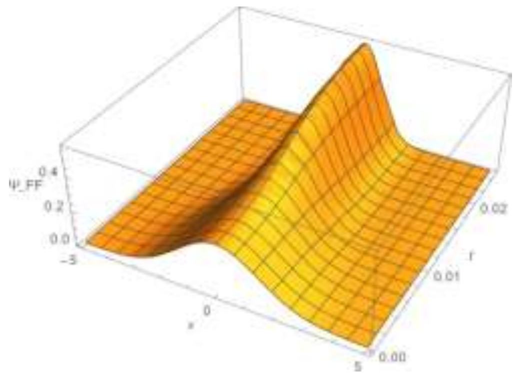

Figure 1. The graph of the complex wave function of the harmonic oscillator

Equation (13) shows that the adiabatic parameter R is related to the change in frequency (ω), thus equation (13) can be rewritten as:

$$\frac{2m}{\hbar}\dot{R}\ Re\left[\phi_n\ \ \frac{\partial\phi_n}{\partial\omega}\right] + 2\ Re\left[{\phi_n}^*\frac{\partial\phi_n}{\partial x}\right]\frac{d\theta}{dx} + \ |\phi_n|^2\frac{\partial^2\theta}{\partial x^2} = 0 \tag{14}$$

Solving equation (14) yields:

$$\frac{2m}{\hbar}\dot{\omega}\left\{\frac{\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\exp[-\frac{m\omega}{\pi\hbar}x^2]}{\frac{\partial\ \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\exp[-\frac{m\omega}{\pi\hbar}x^2}{\partial\omega}}\right\}+2\left\{\frac{\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\exp[-\frac{m\omega}{\pi\hbar}x^2]}{\frac{\partial\ \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\exp[-\frac{m\omega}{\pi\hbar}x^2}{\partial\omega}}\right\}\frac{d\theta}{dx} + \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}\left|\exp[-\frac{m\omega}{\pi\hbar}x^2]\right|^2\frac{\partial^2\theta}{\partial x^2}=0 \tag{15}$$

Thus, we obtain:

$$\frac{d\theta}{dx}=\frac{\hbar}{2xm\omega}\frac{\partial^2\theta}{\partial x^2}-\frac{xm}{2\hbar\omega}\dot{\omega}+\frac{\dot{\omega}}{4x\omega^2} \tag{16}$$

By assuming that the solution θ is a function dependent on x^2, or $\theta = \Phi x^2$, we can derive the following:

$$2\Phi_n x=\frac{\hbar}{2xm\omega}\,2\Phi_n x-\frac{xm}{2\hbar\omega}\dot{\omega}+\frac{\dot{\omega}}{4x\omega^2} \tag{17}$$

$$\frac{\hbar\Phi_n x}{xm\omega}-\frac{xm}{2\hbar\omega}\dot{\omega}+\frac{\dot{\omega}}{4x\omega^2}-2\Phi_n x=0 \tag{18}$$

By grouping together equations (22) that have the same temperature profile,

$$\frac{\hbar\Phi_n x}{xm\omega}+\frac{\dot{\omega}}{4x\omega^2}=0 \tag{19}$$

$$\Phi_n x=-\frac{m}{4\omega\hbar}\dot{\omega} \tag{20}$$

By applying the same procedure to the remaining terms, weobtain:

$$-\frac{xm}{2\hbar\omega}\dot{\omega}-2\Phi_n x=0 \tag{21}$$

$$\Phi_n=-\frac{m}{4\hbar\omega}\dot{\omega} \tag{22}$$

From equations (20) and (22), by substituting the value $\theta = Ax^2$, weobtain the value: The adiabatic phase

$$\theta=-\frac{m\,x^2}{4\omega\hbar}\dot{\omega} \tag{23}$$

Thus, the adiabatic wave function in the *ground state* is given by:

$$\Psi_{0(x,R(t))}=\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}}e^{-\frac{m\omega}{2\hbar}x^2}e^{i-\frac{mx^2}{4\hbar\omega}\dot{\omega}(x,t)} \tag{24}$$

3.2. Driving Potential in Complex Wave Function

To obtain the value of the additional potential, equation (12) is substituted into equation (11), resulting in:

$$\frac{\hbar}{m}\left[\phi_n{}^{*}\frac{\partial \phi_n}{\partial x}\right]\frac{d\theta}{dx} - \frac{\tilde{v}}{\hbar}|\phi_n|^2 + \phi_n{}^{*}\frac{\partial \phi_n}{\partial \mathrm{R}} + \frac{\mathrm{d}\theta}{\mathrm{dt}}|\phi_n|^2 + \frac{\hbar}{2m}|\phi_n|^2\left(\frac{d\theta}{dx}\right)^2 = 0 \quad (25)$$

$$\frac{\hbar}{m}\left[e^{-i\eta}\frac{\partial\, e^{i\eta}}{\partial x}\right]\frac{d\theta}{dx} - \frac{\tilde{v}}{\hbar}\left|e^{i\eta}\right|^2 + e^{-i\eta}\frac{\partial e^{i\eta}}{\partial \mathrm{R}} + \frac{\mathrm{d}\theta}{\mathrm{dt}}\left|e^{i\eta}\right|^2 + \frac{\hbar}{2m}\left|e^{i\eta}\right|^2\left(\frac{d\theta}{dx}\right)^2 = 0 \quad (26)$$

By simplifying equation (26), we obtain:

$$\frac{\hbar}{m}\left[\frac{\partial \eta}{\partial x}\frac{d\theta}{dx}\right] - \frac{\tilde{v}}{\hbar} + \left[\frac{\partial \eta}{\partial \mathrm{R}}\right] + \frac{\mathrm{d}\theta}{\mathrm{dt}} + \frac{\hbar}{2m}\left(\frac{d\theta}{dx}\right)^2 = 0 \quad (27)$$

Thus, the value of the driving potential is obtained as:

$$-mB\int_0^x x^2 \exp\left[\frac{m\omega}{\hbar}x^2\right] dx + \frac{\hbar}{2\omega}\dot{\omega}\,Bx\,\exp\left[\frac{m\omega}{\hbar}x^2\right] dx + \left(\frac{1}{\hbar} - \frac{1}{2\omega}\right)\frac{x^2 m\dot{\omega}^2}{4\omega^2} \quad (28)$$

4. CONCLUSION

The adiabatic phase and additional potential have been obtained to accelerate adiabatic quantum dynamics in complex wave functions. The adiabatic phase and additional potential wereobtained using the fast forward scheme proposed by Masuda and Nakamura in 2010. In this study, a general adiabatic parameter is used, allowing for the useof any arbitrary adiabatic parameter R.Thus, it can be applicable to all time-dependent functions. Theobtained adiabatic phase and additional potential depend on the time derivativeof the frequency of the harmonic oscillator.

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